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ARCS in the Projective Plane Over Some Finite Fields

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Abstract
The investigation of arcs in finite projective planes focuses on analyzing their sizes, structural characteristics, and their connections with other geometric configurations. Arcs can be classified according to several features, including their cardinality and degree. This work examines arcs in the finite projective spaces $PG(2,q)$ for $q = 25, 27$. The study is based on the orbits obtained from the action of the projective linear groups $PGL(3,25)$ and $PGL(3,27)$ on $PG(2,25)$ and $PG(2,27)$, respectively. Furthermore, each obtained arc is extended according to its degree until a complete arc is achieved.

Keyword: ARC, Complete ARCS, Finite field, Finite projective spaces, Vector space

Introduction

The study of arcs in projective spaces on finite fields is characterized by features related to their size (number of points), algebraic structure, and relationship to geometric concepts. The geometric and harmonic study of arcs is one of the most important frameworks, as it can be directly linked to linear error correcting codes. The search for the largest possible arc means searching for the most efficient code for data transmission and errors correcting. Arcs in projection spaces are subject to constraints regarding the number of points (k), the order of the finite field (q), and the dimensions of the space (n). A rational inflexion of a cubic curve $\mathcal{F}$ is a non-singular (simple) point at which the tangent has three points contact. There are non-singular cubic curves formed as a $(k;3)$-arc over a finite field. In this paper classified the cubic curves of order twenty-five and twenty-seven and determined if they are complete or incomplete as arc of degree three. Also, the size of the largest arc of degree three that can be constructed from each incomplete cubic curve is given. Every subject in mathematics has a history. Beniamino Segre is widely considered the first to introduse and study the concept of finite arcs in the projective plane 1955 [6], S. Balland A. Blokhuis and Mazzocco 1997 [15] gave an important result that maximum arcs cannot exist in a plane of odd order, Hirschfeld 1988 [13] explained that the number of projectively distinct arc is hard to calculate. Many studies have addressed arcs in certain finite fields [4, 9, 10, 11, 12]. These studies then evolved to include the relationship between the concept of arcs and the linear codes of various types, general codes in [7, 8], MDS codes in [1], and optimal codes in [6], and their cryptography to network in [21]. Second degree arcs have been classified in the projective plane by many researchers [2, 3]. However, for the arcs with degrees higher than two, they are not fully classified, the third-order arcs [14]. Some researchers have resorted to partial classifications [18, 20].

A line ℓ of $PG(n,q), n > 1$ is said to be " i-secant "of an $(k;r)$-arc K if $|\ell \cap K| = i$.

A 2-secant is called a "*bisecant* ", a 1-secant is a" *unisecant* (*tangent*)" and a 0-secant is an "*external line*": here τ_i is the total number of i-secants to$(k;r)$-arc K. The goal of this research is to present many complete and incomplete arcs in the projective plane $PG(2,25)$ and $PG(2,27)$ and find some of their properties, like T_i-distributions and c_i-distribution. Where the idea of the action of subgroups on space was used (as in [16, 17]). For theoretical details of $PG(n, q), n \geq 1$ see [13].

2. Arcs in a projective space

Definition 4.1[13]: A collection K of k elements of $PG(n,q)$ is called $(k;r)$-arc or arc of degree r with $k \geq r+1$ if every hyperplane incident with at most $r \leq q$ points of K and there is some hyperplane incident with K in exactly r points.

Definition 4.2[13]: An $(k;r)$-arc K is said to be complete if it admits no extension to a larger arc $K'$of the same degree. The maximum value that an arc of degree r will denoted y $m_r(n,q)$.

3. Arcs in the plane [13]

An $(k;r)$-arc in $PG(2,q)$ in the depending on τ_i will be the following:
(1) $\tau_i \geq 0$ for $i < r$;
(2) $\tau_r > 0$
(3) $\tau_i = 0$ for $i > r$.

According to the terminology compete arc of degree r with respect to i-secants the definition will be as following: an $(k;r)$-arc K is complete if every point out K covered by some r-secant of K.

In $PG(2,q)$, the number of copies of a $(k;r)$-arc with stabilizer group G is

$q^3(q^3-1)(q^2-1)/|G|$.

Let π denote any projective plane and τ_i be the total number of the i- secants of (k,r) −arc κ in π then the type of κ denoted by $(\tau_r, \tau_{r-1}, \ldots \tau_0)$. If $(k_1;r)$-arc κ_1 is of the type $(\tau_r, \tau_{r-1}, \ldots \tau_0)$ and $(k_2;\ r)$-arc κ_2 is of the type $(\xi_r, \xi_{r-1}, \ldots, \xi_0)$, then κ_1 and κ_2 have the same type if and only if $\tau_i = \xi_i$ for all i, in this case they are projetively equivalent, let $\sigma_i = \sigma_i(Q)$ the number of i- secant through a point Q of $\pi \setminus \kappa$; the number $\sigma_r(Q)$ of r- secant is the index of Q with respect to κ. Consider c_i be the number of points of $\pi \setminus \kappa$ of index exactly i and C_i be the points of index i.

Theorem 3.1[13]: For $(k;\ r)$-arc κ, the following equations hold:
1. $\sum_{i=0}^{r} \tau_i = q^2 + q + 1$;
2. $\sum_{i=1}^{r} i\tau_i = k(q+1)$
3. $\sum_{i=2}^{r} \frac{i(i-1)\tau_i}{2} = \frac{k(k-1)}{2}$
4. $\sum_{i=1}^{r} \sigma_i = q+1$;
5. $\sum_{i=1}^{r} i\sigma_i = k$;
6. $\sum_{i=1}^{r} (q+1-i)\ \tau_i = \sum_{Q\in\setminus\kappa} \sigma_i$.

4. Projectively distinct curves in $PG(2,25)$ [17]:

Let $\mathcal{F}$ denote the set of points on the cubic curve F'_i, $|\mathcal{F}|$ is the number of points on the curve, $M(\mathcal{F})$ is the size of complete curve formed from $\mathcal{F}(F'_i)$. Suppose that α is the primitive element of F_{25}

Table 1: Non-singular cubic curves parameters of $PG(2,25)$

F'_i	**Coefficient of** $X_0{}^3 + X_1{}^3 X_2{}^3 - bX_0X_1X_2$	$\|\mathcal{F}\| - M(\mathcal{F})$	F'_i	**Coefficients of** $X_2{}^2X_1 + X_0{}^3 + bX_0{}^2X_1 + cX_1{}^3$	$\|\mathcal{F}\| - M(\mathcal{F})$
F'_1	α^{21}	18-36	F'_{41}	α, α^8	28
F'_2	α^6	27	F'_{42}	α, α^6	28
F'_3	0	36	F'_{43}	α^7, α	29
F'_i	$X_0X_1X_2 + bX^3$		F'_{44}	α^7, α^{13}	29
F'_4	α^9	18-31	F'_{45}	$1, \alpha^{22}$	32
F'_5	α^{21}	18-31	F'_{46}	$1, \alpha^6$	32
F'_i	Coefficient of $X_2{}^2X_1 + X_0{}^3 - bX_1{}^3$		F'_{47}	$1, \alpha^{14}$	32
F'_6	α^2	21-36	F'_{48}	α^7, α^4	34
F'_7	α^4	21-36	F'_{49}	α^7, α^{14}	34
F'_i	Coefficient of $X_0X_1X_2 + bX^3$		F'_{50}	$\alpha^{22}, 1$	34
F'_8	α	24	F'_{51}	$0, \alpha^9$	16-32
F'_9	α^7	24	F'_{52}	$\alpha^{22}, 0$	20-28
F'_{10}	α^{22}	24	F'_{53}	$0, \alpha$	31
F'_{11}	α^5	24	F'_{54}	$0, \alpha^7$	31
F'_{12}	α^{14}	24	F'_{55}	$1, 0$	32
F'_{13}	α^{11}	24	F'_{56}	$\alpha^7, 0$	34
F'_{14}	α^{15}	27	F'_i	Coefficient of $X_0{}^3 + aX_1{}^3 + bX_2{}^3 - c\,X_0X_1X_2$	
F'_{15}	α^3	27	F'_{57}	$\alpha, \alpha^2, \alpha^6$	18-30
F'_{16}	α^{19}	30	F'_{58}	$\alpha, \alpha^2, \alpha^{22}$	18-30
F'_{17}	α^4	30	F'_{59}	α, α^2, α	27
F'_{18}	α^{20}	30	F'_{60}	$\alpha, \alpha^2, \alpha^7$	27
F'_{19}	α^{23}	30	F'_{61}	$\alpha, \alpha^2, \alpha^{16}$	27

F'_{20}	α^{10}	33	F'_{62}	$\alpha, \alpha^2, \alpha^4$	27
F'_{21}	α^2	33	F'_{63}	$\alpha, \alpha^2, 0$	36
F'_i	Coefficients of ${X_2}^2X_1 + {X_0}^3 + b{X_0}^2X_1 + c{X_1}^3$		F'_{64}	$\alpha, \alpha^2, \alpha^{13}$	36
F'_{22}	$1, \alpha^{15}$	17-32	F'_i	Coefficients of ${X_0}^2X_1 + {X_0}^2X_2 + a\,{X_1}^2X_2 - b(\,{X_0}^3 + a\,{X_1}^3 + a^2{X_2}^3 - c\,X_0X_1X_2\,)$	
F'_{23}	$\alpha^7, 1$	19-31	F'_{65}	$\alpha, \alpha^{10}, \alpha^{19}$	18-32
F'_{24}	α^7, α^6	19-31	F'_{66}	$\alpha^2, \alpha^4, \alpha^{20}$	18-32
F'_{25}	α^{22}, α^7	20-28	F'_{67}	$\alpha, 0$	21
F'_{26}	α^{22}, α^{19}	20-30	F'_{68}	$\alpha^2, 0$	21
F'_{27}	α^{22}, α^{15}	20-28	F'_{69}	$\alpha, \alpha^7, \alpha^{19}$	24
F'_{28}	$1, \alpha$	22-27	F'_{70}	$\alpha, \alpha^{13}, \alpha^{19}$	24
F'_{29}	$1, \alpha^{13}$	22-25	F'_{71}	$\alpha, 1, \alpha^{19}$	24
F'_{30}	$1, \alpha^8$	22-27	F'_{72}	$\alpha^2, \alpha^{13}, \alpha^{20}$	24
F'_{31}	$1, \alpha^5$	22-25	F'_{73}	$\alpha^2, \alpha^{19}, \alpha^{20}$	24
F'_{32}	α, α^4	23-26	F'_{74}	$\alpha^2, \alpha^{10}, \alpha^{20}$	24
F'_{33}	α, α^{14}	23-26	F'_{75}	$\alpha, \alpha^{19}, \alpha^{19}$	27
F'_{34}	α^{22}, α^8	25	F'_{76}	$\alpha^2, \alpha, \alpha^{20}$	27
F'_{35}	α^{22}, α^9	25-28	F'_{77}	$\alpha, \alpha, \alpha^{19}$	30
F'_{36}	α^{22}, α^4	25	F'_{78}	$\alpha, \alpha^{22}, \alpha^{19}$	30
F'_{37}	α, α	28	F'_{79}	$\alpha^2, 1, \alpha^{20}$	30
F'_{38}	α, α^{13}	28	F'_{80}	$\alpha^2, \alpha^7, \alpha^{20}$	30
F'_{39}	$\alpha, 1$	28	F'_{81}	$\alpha, \alpha^4, \alpha^{19}$	30
F'_{40}	α, α^{22}	28	F'_{82}	$\alpha^2, \alpha^{22}, \alpha^{20}$	30

5. Arcs in $PG(2, 25)$

From the Theorem 3.1and using GAP Programming system, the following results are deduced:

Corollary5.1: Over F_{25},

There are 108 arcs of degree three computed from the non- singular cubic curves have the secant distributions, τ_i and c_i given in the following table.

Table 2: Full details of (k;3)-arcs in $PG(2,25)$

F	$[\tau_0, \tau_1, \tau_2, \tau_3]$	$[c_0, c_1, \ldots, c_m]$
F_1	[288, 306, 9, 48]	[45, 180, 300,108]
Extension of F_1	[162, 225, 81, 183]	[18, 27, 153, 288, 102, 27]
F_2	[189, 333, 18, 111]	[24, 126, 288,141, 45]
F_3	[144, 279, 27, 201]	[81, 252, 171,108, 3]
F_4	[290, 300, 15, 46]	[42, 221, 282,79, 9]
Extension of F_4	[202, 200, 141, 108]	[14, 55, 135,204,145, 57, 8, 2]
F_5	[290, 300, 15, 46]	[42, 221, 282,79, 9]
Extension of F_4	[202, 200, 141,108]	[14, 55, 135, 204, 145, 57, 8, 2]
F_6	[251, 318, 18, 64]	[21, 60, 276,240, 30, 1, 2]
Extension of F_6	[180, 171, 135, 165]	[3, 69, 126,144, 192, 60, 18, 3]
F_7	[251, 318, 18, 64]	[21, 60, 276,240, 30, 1, 2]
Extension of F_7	[180, 171, 135, 165]	[3, 69, 126,144, 192, 60, 18, 3]
F_8	[218, 327, 21, 85]	[21, 135, 253,191, 21, 6]
F_9	[218, 327, 21, 85]	[21, 126, 280,161, 36, 3]
F_{10}	[218, 327, 21, 85]	[4, 9, 153,237, 191, 33]
Extension of F_{10}	[198, 281, 69, 103]	[72, 153, 242,138, 18]
F_{11}	[218, 327, 21, 85]	[21, 135, 253,191, 21, 6]
F_{12}	[218, 327, 21, 85]	[4, 9, 153,237, 191, 33]
Extension of F_{12}	[198, 281, 69, 103]	[72, 153, 242,138, 18]
F_{13}	[218, 327, 21, 85]	[21, 126, 280,161, 36, 3]
F_{14}	[189, 333, 18, 111]	[24, 126, 288,141, 45]
F_{15}	[191, 327, 24, 109]	[3, 30, 132,287, 132, 40]
F_{16}	[170, 318, 27, 136]	[6, 31, 120,273, 159, 32]
F_{17}	[170, 318, 27, 136]	[27, 164, 222,178, 27, 3]
F_{18}	[170, 318, 27, 136]	[27, 164, 222,178, 27, 3]
F_{19}	[170, 318, 27, 136]	[6, 31, 120,273, 159, 32]
F_{20}	[155, 300, 30, 166]	[3, 29, 102,249, 195, 36, 4]
F_{21}	[155, 300, 30, 166]	[3, 29, 102,249, 195, 36, 4]
F_{22}	[305, 290, 16, 40]	[71, 264, 242,56, 1]
Extension of F_{22}	[192, 209, 127, 123]	[3, 23, 100,167, 183, 107, 33, 3]
F_{23}	[277, 305, 18, 51]	[33, 172, 294,119, 14]

Extension of F_{23}	[192, 230, 111, 118]	[4, 33, 104,185, 192, 82, 20]
F_{24}	[277, 305, 18, 51]	[33, 172, 294,119, 14]
Extension of F_{24}	[187, 245, 96, 123]	[2, 17, 89,199, 180, 102, 30, 1]
F_{25}	[264, 311, 19, 57]	[20, 128, 296,158, 28, 1]
Extension of F_{25}	[208, 251, 99, 93]	[22, 99, 201,208, 74, 19]
F_{26}	[264, 311, 19, 57]	[16, 140, 280,170, 24, 1]
Extension of F_{26}	[204, 216, 129, 102]	[11, 78, 158,219, 109, 39, 6, 1]
F_{27}	[264, 311, 19, 57]	[20, 128, 296,158, 28, 1]
Extension of F_{27}	[208, 251, 99, 93]	[22, 99, 201,208, 74, 19]
F_{28}	[240, 320, 21, 70]	[5, 73, 204,265, 76, 6]
Extension of F_{28}	[208, 276, 75, 92]	[18, 98, 221,210, 64, 12, 1]
F_{29}	[240, 320, 21, 70]	[3, 72, 217,252, 77, 8]
Extension of F_{29}	[222, 287, 63, 79]	[39, 172, 259,125, 29, 2]
F_{30}	[240, 320, 21, 70]	[5, 73, 204,265, 76, 6]
Extension of F_{30}	[208, 276, 75, 92]	[18, 98, 221,210, 64, 12, 1]
F_{31}	[240, 320, 21, 70]	[3, 72, 217,252, 77, 8]
Extension of F_{31}	[222, 287, 63, 79]	[39, 172, 259,125, 29, 2]
F_{32}	[229, 323, 22, 77]	[5, 37, 176,275, 120, 13, 2]
Extension of F_{32}	[212, 290, 61, 88]	[11, 124, 259,171, 56, 4]
F_{33}	[229, 323, 22, 77]	[5, 37, 176,275, 120, 13, 2]
Extension of F_{33}	[212, 290, 61, 88]	[11, 124, 259,171, 56, 4]
F_{34}	[209, 326, 24, 92]	[10, 99, 231,223, 55, 8]
F_{35}	[209, 326, 24, 92]	[3, 120, 202,239, 58, 4]
Extension of F_{35}	[196, 287, 63, 105]	[44, 180, 242, 275, 123, 34]
F_{36}	[209, 326, 24, 92]	[10, 99, 231,223, 55, 8]
F_{37}	[184, 323, 27, 117]	[13, 110, 232,203, 63, 2]
F_{38}	[184, 323, 27, 117]	[11, 119, 220,211, 57, 5]
F_{39}	[184, 323, 27, 117]	[17, 105, 229,208, 62, 2]
F_{40}	[184, 323, 27, 117]	[17, 105, 229,208, 62, 2]
F_{41}	[184, 323, 27, 117]	[11, 119, 220,211, 57, 5]
F_{42}	[184, 323, 27, 117]	[13, 110, 232, 203, 63, 2]
F_{43}	[177, 320, 28, 126]	[6, 63, 203, 227, 111, 12]
F_{44}	[179, 310, 46, 112, 4]	[1, 1, 27, 138, 235, 158, 57, 5]
F_{45}	[160, 305, 31, 55]	[7, 54, 162, 281, 89, 25, 1]
F_{46}	[160, 305, 31, 55]	[6, 53, 175, 252, 116, 17]
F_{47}	[160, 305, 31, 55]	[7, 54, 162, 281, 89, 25, 1]
F_{48}	[152, 290, 33, 176]	[13, 69, 206, 226, 94, 9]
F_{49}	[152, 290, 33, 176]	[13, 69, 206, 226, 94, 9]
F_{50}	[149, 281, 34, 187]	[4, 37, 147, 240, 154, 34]
F_{51}	[320, 281, 15, 35]	[96, 302, 210, 26, 1]
Extension of F_{51}	[196, 197, 139, 119]	[3, 35, 102, 196, 161, 89, 30, 2, 1]
F_{52}	[264, 311, 19, 57]	[16, 135, 296, 152, 32]
Extension of F_{52}	[212, 239, 111, 89]	[26, 129, 214, 169, 15, 20]
F_{53}	[165, 311, 30, 145]	[16, 102, 207, 224, 70, 1]
F_{54}	[165, 311, 30, 145]	[16, 102, 207, 224, 70, 1]
F_{55}	[160, 305, 31, 155]	[72, 141, 300, 76, 30]
F_{56}	[152, 290, 33, 176]	[4, 88, 188, 236, 93, 8]
F_{57}	[291, 297, 18, 45]	[39, 252, 252, 81, 9]
Extension of F_{57}	[201, 225, 120, 105]	[12, 54, 180, 183, 138, 45, 9]
F_{58}	[291, 297, 18, 45]	[39, 252, 252, 81, 9]
Extension of F_{58}	[201, 225, 120, 105]	[8, 66, 163, 198, 135, 39, 11, 1]
F_{59}	[291, 297, 18, 45]	[39, 255, 243, 90, 6]
Extension of F_{59}	[194, 203, 133, 121]	[5, 24, 103, 176, 182, 96, 33]
F_{60}	[291, 297, 18, 45]	[39, 255, 243, 90, 6]
Extension of F_{60}	[193, 206, 130, 122]	[4, 22, 100, 183, 177, 94, 35, 4]
F_{61}	[252, 315, 21, 63]	[108, 306, 135, 81]
F_{62}	[252, 315, 21, 63]	[108, 306, 135, 81]
F_{63}	[219, 324, 24, 84]	[24, 144, 252, 171, 36]
F_{64}	[219, 324, 24, 84]	[33, 111, 291, 156, 36]
F_{65}	[219, 324, 24, 84]	[15, 168, 234, 171, 39]
F_{66}	[219, 324, 24, 84]	[24, 144, 252, 171, 36]
F_{67}	[219, 324, 24, 84]	[33, 111, 291, 156, 36]
F_{68}	[219, 324, 24, 84]	[15, 168, 234, 171, 39]
F_{69}	[192, 324, 27, 108]	[18, 201, 216, 162, 18, 9]
F_{70}	[192, 324, 27, 108]	[9, 18, 174, 216, 189, 18]
F_{71}	[192, 324, 27, 108]	[9, 27, 144, 252, 171, 21]
F_{72}	[192, 324, 27, 108]	[9, 27, 144, 252, 171, 21]
F_{73}	[192, 324, 27, 108]	[3, 30, 153, 264, 141, 30, 3]

F_{74}	[192, 324, 27, 108]	[3, 30, 153, 264, 141, 30, 3]
F_{75}	[171, 315, 30, 135]	[6, 33, 126, 282, 141, 30, 3]
F_{76}	[171, 315, 30, 135]	[3, 39, 132, 261, 153, 33]
F_{77}	[171, 315, 30, 135]	[6, 33, 126, 282, 141, 30, 3]
F_{78}	[171, 315, 30, 135]	[3, 39, 132, 261, 153, 33]
F_{79}	[156, 297, 33, 165]	[6, 15, 138, 234, 183, 36, 6]
F_{80}	[156, 297, 33, 165]	[6, 15, 138, 234, 183, 36, 6]
F_{81}	[147, 270, 36, 198]	[9, 99, 180, 297, 18, 9, 3]
F_{82}	[147, 270, 36, 198]	[18, 75, 252, 189, 72, 9]

6. Projectively distinct curves in $PG(2, 27)$ [16]:

Table 3: Cubic curves with exactly three rational inflexions

Canonical form $\mathcal{F} = v(F')$	$\lvert\mathcal{F}\rvert$	Description	$M(\mathcal{F})$	G
$F'_1 = X_0X_1X_2 + (X_0 + X_1 + X_2)^3$	18	incomplete	30	S_3
$F'_2 = X_0X_1X_2 + \beta^{21}(X_0 + X_1 + X_2)^3$	21	incomplete	33	S_3
$F'_3 = X_0X_1X_2 + \beta^{11}(X_0 + X_1 + X_2)^3$	21	incomplete	33	S_3
$F'_4 = X_0X_1X_2 + \beta^{7} (X_0 + X_1 + X_2)^3$	21	incomplete	33	S_3
$F'_5 = X_0X_1X_2 + \beta^{15} (X_0 + X_1 + X_2)^3$	24	complete	–	S_3
$F'_6 = X_0X_1X_2 + \beta^{20} (X_0 + X_1 + X_2)^3$	24	incomplete	25	S_3
$F'_7 = X_0X_1X_2 + \beta^{24} (X_0 + X_1 + X_2)^3$	24	incomplete	25	S_3
$F'_8 = X_0X_1X_2 + \beta^{8} (X_0 + X_1 + X_2)^3$	24	incomplete	25	S_3
$F'_9 = X_0X_1X_2 + \beta^{5}(X_0 + X_1 + X_2)^3$	24	complete	–	S_3
$F'_{10} = X_0X_1X_2 + \beta^{19} (X_0 + X_1 + X_2)^3$	24	complete	–	S_3
$F'_{11} = X_0X_1X_2 + \beta (X_0 + X_1 + X_2)^3$	27	complete	–	S_3
$F'_{12} = X_0X_1X_2 + \beta^{9} (X_0 + X_1 + X_2)^3$	27	complete	–	S_3
$F'_{13} = X_0X_1X_2 + \beta^{3} (X_0 + X_1 + X_2)^3$	27	complete	–	S_3
$F'_{14} = X_0X_1X_2 + \beta^{2} (X_0 + X_1 + X_2)^3$	30	complete	–	S_3
$F'_{15} = X_0X_1X_2 + \beta^{10} (X_0 + X_1 + X_2)^3$	30	complete	–	S_3
$F'_{16} = X_0X_1X_2 + \beta^{4} (X_0 + X_1 + X_2)^3$	30	complete	–	S_3
$F'_{17} = X_0X_1X_2 + \beta^{6} (X_0 + X_1 + X_2)^3$	30	complete	–	S_3
$F'_{18} = X_0X_1X_2 + \beta^{18} (X_0 + X_1 + X_2)^3$	30	complete	–	S_3
$F'_{19} = X_0X_1X_2 + \beta^{12} (X_0 + X_1 + X_2)^3$	30	complete	–	S_3
$F'_{20} = X_0X_1X_2 + \beta^{17} (X_0 + X_1 + X_2)^3$	33	complete	–	S_3
$F'_{21} = X_0X_1X_2 + \beta^{23} (X_0 + X_1 + X_2)^3$	33	complete	–	S_3
$F'_{22} = X_0X_1X_2 + \beta^{25} (X_0 + X_1 + X_2)^3$	33	complete	–	S_3
$F'_{23} = X_0X_1X_2 + \beta^{14} (X_0 + X_1 + X_2)^3$	36	complete	–	S_3
$F'_{24} = X_0X_1X_2 + \beta^{16} (X_0 + X_1 + X_2)^3$	36	complete	–	S_3
$F'_{25} = X_0X_1X_2 + \beta^{13} (X_0 + X_1 + X_2)^3$	36	complete	–	S_3
$F'_{26} = X_0X_1X_2 + \beta^{22} (X_0 + X_1 + X_2)^3$	36	complete	–	S_3

Table 4: Cubic curves with exactly one inflexion of type G

Canonical form	$\lvert\mathcal{F}\rvert$	Description	$M(\mathcal{F})$	G
$F'_{27} = X_1 + X_0^3 + \beta^2 X_0^2 X_1 + \beta^2 X_1^3$	20	incomplete	32	Z_2
$F'_{28} = X_2^2X_1 + X_0^3 + \beta^2 X_0^2 X_1 + \beta^{20} X_1^3$	20	incomplete	32	Z_2
$F'_{29} = X_2^2X_1 + X_0^3 + \beta^2 X_0^2 X_1 + \beta^{22} X_1^3$	20	incomplete	32	Z_2
$F'_{30} = X_2^2X_1 + X_0^3 + \beta^2 X_0^2 X_1 + \beta^{19} X_1^3$	20	incomplete	30	Z_2
$F'_{31} = X_2^2X_1 + X_0^3 + \beta^2 X_0^2 X_1 + \beta^{23} X_1^3$	23	incomplete	29	Z_2
$F'_{32} = X_2^2X_1 + X_0^3 + \beta^2 X_0^2 X_1 + \beta^{3} X_1^3$	23	incomplete	29	Z_2
$F'_{33} = X_2^2X_1 + X_0^3 + \beta^2 X_0^2 X_1 + \beta^{5} X_1^3$	23	incomplete	29	Z_2
$F'_{34} = X_2^2X_1 + X_0^3 + \beta^2 X_0^2 X_1 + \beta^{10} X_1^3$	26	incomplete	27	Z_2
$F'_{35} = X_2^2X_1 + X_0^3 + \beta^2 X_0^2 X_1 + \beta^{16} X_1^3$	26	incomplete	27	Z_2
$F'_{36} = X_2^2X_1 + X_0^3 + \beta^2 X_0^2 X_1 + \beta^{18} X_1^3$	26	incomplete	27	Z_2
$F'_{37} = X_2^2X_1 + X_0^3 + \beta^2 X_0^2 X_1 + \beta^{24} X_1^3$	26	incomplete	28	Z_2
$F'_{38} = X_2^2X_1 + X_0^3 + \beta^2 X_0^2 X_1 + \beta^{12} X_1^3$	26	incomplete	28	Z_2
$F'_{39} = X_2^2X_1 + X_0^3 + \beta^2 X_0^2 X_1 + \beta^{8} X_1^3$	26	incomplete	28	Z_2
$F'_{40} = X_2^2X_1 + X_0^3 + \beta^2 X_0^2 X_1 + \beta^{15} X_1^3$	29	complete	–	Z_2
$F'_{41} = X_2^2X_1 + X_0^3 + \beta^2 X_0^2 X_1 + \beta^{9} X_1^3$	29	complete	–	Z_2
$F'_{42} = X_2^2X_1 + X_0^3 + \beta^2 X_0^2 X_1 + \beta^{7} X_1^3$	29	complete	–	Z_2
$F'_{43} = X_2^2X_1 + X_0^3 + \beta^2 X_0^2 X_1 + \beta^{14} X_1^3$	32	complete	–	Z_2
$F'_{44} = X_2^2X_1 + X_0^3 + \beta^2 X_0^2 X_1 + \beta^{4} X_1^3$	32	complete	–	Z_2
$F'_{45} = X_2^2X_1 + X_0^3 + \beta^2 X_0^2 X_1 + X_1^3$	32	complete	–	Z_2
$F'_{46} = X_2^2X_1 + X_0^3 + \beta^2 X_0^2 X_1 + \beta^{21} X_1^3$	32	complete	–	Z_2
$F'_{47} = X_2^2X_1 + X_0^3 + \beta^2 X_0^2 X_1 + \beta^{25} X_1^3$	32	complete	–	Z_2
$F'_{48} = X_2^2X_1 + X_0^3 + \beta^2 X_0^2 X_1 + \beta^{11} X_1^3$	32	complete	–	Z_2

$F'_{49} = X_2^2X_1 + X_0^3 + \beta^2 X_0^2 X_1 + \beta X_1^3$	35	complete	−	Z_2
$F'_{50} = X_2^2X_1 + X_0^3 + \beta^2 X_0^2 X_1 + \beta^{17} X_1^3$	35	complete	−	Z_2
$F'_{51} = X_2^2X_1 + X_0^3 + \beta^2 X_0^2 X_1 + \beta^{13} X_1^3$	35	complete	−	Z_2
$F'_{52} = X_2^2X_1 + X_0^3 + \beta^2 X_0^2 X_1 + \beta^6 X_1^3$	38	complete	−	Z_2

Table 5: Non-singular cubic curves with exactly one inflexion of type S

Canonical form	$\lvert\mathcal{F}\rvert$	Description	$M(\mathcal{F})$	G
$\bar{F}_{53} = X_2^2X_1 + X_0^3 + \beta^{13} X_0 X_1^2 + \beta^2 X_1^3$	19	incomplete	30	Z_6
$\bar{F}_{54} = X_2^2X_1 + X_0^3 + \beta^{14} X_0 X_1^2$	28	incomplete	29	Z_2
$\bar{F}_{55} = X_2^2X_1 + X_0^3 + \beta^{13} X_0 X_1^2$	28	complete	_	Z_6
$\bar{F}_{56} = X_2^2X_1 + X_0^3 + \beta^{13} X_0 X_1^2 + \beta^{15} X_1^3$	37	complete	−	Z_6

Table 6: Cubic curves with no rational inflexions

Canonical form	$\lvert\mathcal{F}\rvert$	Description	$M(\mathcal{F})$	G
$F'_{57}=X_o^3+X_1^3 + \beta^{23}X_2^3 + \beta^{15}X_0^2X_2 + \beta^{15}X_0 X_1^2+\beta^4X_0 X_2^2+\beta^{15}X_1 X_2^2$	18	incomplete	34	Z_3
$F'_{58}=X_o^3+X_1^3 + \beta^{16}X_2^3 + \beta^{15}X_0^2X_2 + \beta^{15}X_0 X_1^2 + \beta^4X_0 X_2^2+\beta^{15}X_1 X_2^2$	21	incomplete	30	Z_3
$F'_{59}=X_o^3+X_1^3 + \beta^{18}X_2^3 + \beta^{15}X_0^2X_2 + \beta^{15}X_0 X_1^2 + \beta^4X_0 X_2^2+\beta^{15}X_1 X_2^2$	21	incomplete	30	Z_3
$F'_{60}=X_o^3+X_1^3 + \beta^{13}X_2^3 + \beta^{15}X_0^2X_2 + \beta^{15}X_0 X_1^2 + \beta^4X_0 X_2^2+\beta^{15}X_1 X_2^2$	21	incomplete	30	Z_3
$F'_{61}=X_o^3+X_1^3 + \beta^{15}X_0^2X_2 + \beta^{15}X_0 X_1^2 + \beta^4X_0 X_2^2+\beta^{15}X_1 X_2^2$	24	incomplete	27	Z_3
$F'_{62}=X_o^3+X_1^3 + \beta^2X_2^3 + \beta^{15}X_0^2X_2 + \beta^{15}X_0 X_1^2 + \beta^4X_0 X_2^2+\beta^{15}X_1 X_2^2$	24	incomplete	27	Z_3
$F'_{63}=X_o^3+X_1^3 + \beta^{15}X_2^3 + \beta^{15} X_0^2 X_2 + \beta^{15}X_0 X_1^2\beta^4+X_0 X_2^2+\beta^{15}X_1 X_2^2$	24	incomplete	27	Z_3
$F'_{64}=X_o^3+X_1^3 + \beta^9X_2^3 + \beta^{15}X_0^2X_2 + \beta^{15}X_0 X_1^2 + \beta^4X_0 X_2^2+\beta^{15}X_1 X_2^2$	24	incomplete	27	Z_3
$F'_{65} = X_o^3+X_1^3 + \beta^{20}X_2^3 + \beta^{15}X_0^2X_2 + \beta^{15}X_0 X_1^2+\beta^4X_0 X_2^2+\beta^{15}X_1 X_2^2$	24	incomplete	27	Z_3
$F'_{66} = X_o^3+X_1^3 + \beta^{19}X_2^3 + \beta^{15}X_0^2X_2 + \beta^{15}X_0X_1^2 +\beta^4X_0 X_2^2+\beta^{15}X_1 X_2^2$	24	incomplete	27	Z_3
$F'_{67} = X_o^3+X_1^3 + \beta^{14}X_2^3 + \beta^{15}X_0^2X_2 + \beta^{15}X_0 X_1^2+\beta^4X_0 X_2^2+\beta^{15}X_1 X_2^2$	27	complete	−	Z_3
$F'_{68} = X_o^3+X_1^3 + \beta^{21}X_2^3 + \beta^{15}X_0^2X_2 + \beta^{15}X_0 X_1^2+\beta^4X_0 X_2^2+\beta^{15}X_1 X_2^2$	27	complete	−	Z_3
$F'_{69} = X_o^3+X_1^3 + \beta^5X_2^3 + \beta^{15}X_0^2X_2 + \beta^{15}X_0 X_1^2+\beta^4X_0 X_2^2+\beta^{15}X_1 X_2^2$	27	complete	−	Z_3
$F'_{70} = X_o^3+X_1^3 + \beta X_2^3 + \beta^{15}X_0^2X_2 + \beta^{15}X_0 X_1^2 + \beta^4X_0 X_2^2+\beta^{15}X_1 X_2^2$	30	complete	−	Z_3
$F'_{71} = X_o^3+X_1^3 + \beta^{10}X_2^3 + \beta^{15}X_0^2X_2 + \beta^{15}X_0 X_1^2+\beta^4X_0 X_2^2+\beta^{15}X_1 X_2^2$	30	complete	−	Z_3
$F'_{72} = X_o^3+X_1^3 + \beta^{17}X_2^3 + \beta^{15}X_0^2X_2 + \beta^{15}X_0 X_1^2+\beta^4X_0 X_2^2+\beta^{15}X_1 X_2^2$	30	complete	−	Z_3
$F'_{73} = X_o^3+X_1^3 + \beta^{24}X_2^3 + \beta^{15}X_0^2X_2 + \beta^{15}X_0 X_1^2+\beta^4X_0 X_2^2+\beta^{15}X_1 X_2^2$	30	complete	−	Z_3
$F'_{74} = X_o^3+X_1^3 + \beta^{12}X_2^3 + \beta^{15}X_0^2X_2 + \beta^{15}X_0 X_1^2+\beta^4X_0 X_2^2+\beta^{15}X_1 X_2^2$	30	complete	−	Z_3
$F'_{75} = X_o^3+X_1^3 + \beta^8X_2^3 + \beta^{15}X_0^2X_2 + \beta^{15}X_0 X_1^2+\beta^4X_0 X_2^2+\beta^{15}X_1 X_2^2$	30	complete	−	Z_3
$F'_{76} = X_o^3+X_1^3 + \beta^6X_2^3 + \beta^{15}X_0^2X_2 + \beta^{15}X_0 X_1^2+\beta^4X_0 X_2^2+\beta^{15}X_1 X_2^2$	33	complete	−	Z_3
$F'_{77} = X_o^3+X_1^3 + \beta^{22}X_2^3 + \beta^{15}X_0^2X_2 + \beta^{15}X_0 X_1^2+\beta^4X_0 X_2^2+\beta^{15}X_1 X_2^2$	33	complete	−	Z_3
$F'_{78} = X_o^3+X_1^3 + \beta^7X_2^3 + \beta^{15}X_0^2X_2 + \beta^{15}X_0 X_1^2+\beta^4X_0 X_2^2+\beta^{15}X_1 X_2^2$	33	complete	−	Z_3
$F'_{79} = X_o^3+X_1^3 + \beta^4X_2^3 + \beta^{15}X_0^2X_2 + \beta^{15}X_0 X_1^2+\beta^4X_0 X_2^2+\beta^{15}X_1 X_2^2$	36	complete	−	S_3
$F'_{80} = X_o^3+X_1^3 + \beta^{25}X_2^3 + \beta^{15}X_0^2X_2 + \beta^{15}X_0 X_1^2+\beta^4X_0 X_2^2+\beta^{15}X_1 X_2^2$	36	complete	−	S_3
$F'_{81} = X_o^3+X_1^3 + \beta^3X_2^3 + \beta^{15}X_0^2X_2 + \beta^{15}X_0 X_1^2+\beta^4X_0 X_2^2+\beta^{15}X_1 X_2^2$	36	complete	−	S_3
$F'_{82} = X_o^3+X_1^3 + \beta^{11}X_2^3 + \beta^{15}X_0^2X_2 + \beta^{15}X_0 X_1^2+\beta^4X_0 X_2^2+\beta^{15}X_1 X_2^2$	36	complete	−	S_3

7. Arcs in $GF(27)$

4.3. Arcs in $PG(2, 27)$

From the Theorem and using GAP Programming system, the following result is deduced:

Corollary 7.1: Over F_{27},
There are 114 arcs of degree three computed from the non- singular cubic curves have the secant distributions, τ_i and c_i given in the following table**:**

Table 7: Full details of $(k,3)$-arcs from cubic curves

F'	$[\tau_0, \tau_1, \tau_2, \tau_3]$	$[c_0, c_1, \dots, c_m]$
F'_1	[360, 336, 15, 46]	[54, 319, 276, 81, 9]
Extension of F'_1	[252, 270, 135, 100]	[21, 137, 238, 207, 88, 32,3,1]
F'_2	[315, 360, 18, 64]	[15, 151, 306, 219, 45]
Extension of F'_2	[231, 258, 138, 130]	[1, 21, 159, 180, 207, 126,21,9]
F'_3	[315, 360, 18, 64]	[15, 151, 306, 219, 45]
Extension of F'_3	[231, 258, 138, 130]	[1, 21, 159, 180, 207, 126,21,9]
F'_4	[315, 360, 18, 64]	[15, 151, 306, 219, 45]
Extension of F'_4	[231, 258, 138, 130]	[1, 21, 159, 180, 207, 126,21,9]
F'_5	[276, 375, 21, 85]	[15, 151, 306, 219, 45]
F'_6	[276, 375, 21, 85]	[3, 28, 225, 297, 144, 36]
Extension of F'_6	[268, 367, 33, 89]	[22, 195, 297, 170, 46, 2]
F'_7	[276, 375, 21, 85]	[3, 28, 225, 297, 144, 36]
Extension of F'_7	[268, 367, 33, 89]	[22, 195, 297, 170, 46, 2]
F'_8	[276, 375, 21, 85]	[3, 28, 225, 297, 144, 36]

Extension of F'_8	[268, 367, 33, 89]	[22, 195, 297, 170, 46, 2]
F'_9	[276, 375, 21, 85]	[51, 177, 327, 154, 21, 3]
F'_{10}	[276, 375, 21, 85]	[51, 177, 327, 154, 21, 3]
F'_{11}	[243, 381, 24, 109]	[3, 90, 156, 351, 114, 12, 4]
F'_{12}	[243, 381, 24, 109]	[3, 90, 156, 351, 114, 12, 4]
F'_{13}	[243, 381, 24, 109]	[3, 90, 156, 351, 114, 12, 4]
F'_{14}	[216, 378, 27, 136]	[1, 6, 57, 267, 255, 114, 27]
F'_{15}	[216, 378, 27, 136]	[15, 72, 192, 315, 120, 13]
F'_{16}	[216, 378, 27, 136]	[15, 72, 192, 315, 120, 13]
F'_{17}	[216, 378, 27, 136]	[1, 6, 57, 267, 255, 114, 27]
F'_{18}	[216, 378, 27, 136]	[1, 6, 57, 267, 255, 114, 27]
F'_{19}	[216, 378, 27, 136]	[15, 72, 192, 315, 120, 13]
F'_{20}	[195, 366, 30, 166]	[3, 9, 60, 210, 282, 145, 15]
F'_{21}	[195, 366, 30, 166]	[3, 9, 60, 210, 282, 145, 15]
F'_{22}	[195, 366, 30, 166]	[3, 9, 60, 210, 282, 145, 15]
F'_{23}	[180, 345, 33, 199]	[6, 54, 174, 295, 156, 36]
F'_{24}	[180, 345, 33, 199]	[6, 54, 174, 295, 156, 36]
F'_{25}	[180, 345, 33, 199]	[1, 51, 207, 258, 168, 36]
F'_{26}	[180, 345, 33, 199]	[6, 54, 174, 295, 156, 36]
F'_{27}	[330, 351, 19, 57]	[30, 194, 333, 155, 25]
Extension of F'_{27}	[242, 249, 151, 115]	[8, 70, 187, 233, 144, 67, 16]
F'_{28}	[330, 351, 19, 57]	[30, 194, 333, 155, 25]
Extension of F'_{28}	[237, 264, 136, 120]	[6, 53, 160, 241, 172, 71, 18, 4]
F'_{29}	[330, 351, 19, 57]	[30, 194, 333, 155, 25]
Extension of F'_{29}	[237, 264, 136, 120]	[6, 53, 160, 241, 172, 71, 18, 4]
F'_{30}	[330, 351, 19, 57]	[32, 188, 335, 161, 21]
Extension of F'_{30}	[247, 285, 120, 105]	[17, 104, 232, 208, 127, 36, 3]
F'_{31}	[289, 369, 22, 77]	[8, 55, 285, 255, 121, 9, 1]
Extension of F'_{31}	[254, 291, 115, 97]	[16, 152, 264, 190, 85, 19, 2]
F'_{32}	[289, 369, 22, 77]	[8, 55, 285, 255, 121, 9, 1]
Extension of F'_{32}	[254, 291, 115, 97]	[16, 152, 264, 190, 85, 19, 2]
F'_{33}	[289, 369, 22, 77]	[8, 55, 285, 255, 121, 9, 1]
Extension of F'_{33}	[254, 291, 115, 97]	[16, 152, 264, 190, 85, 19, 2]
F'_{34}	[254, 378, 225, 100]	[1, 6, 121, 259, 253, 83, 8]
Extension of F'_{34}	[248, 366, 39, 104]	[6, 95, 241, 273, 101, 14]
F'_{35}	[254, 378, 225, 100]	[1, 6, 121, 259, 253, 83, 8]
Extension of F'_{35}	[248, 366, 39, 104]	[6, 95, 241, 273, 101, 14]
F'_{36}	[254, 378, 225, 100]	[1, 6, 121, 259, 253, 83, 8]
Extension of F'_{36}	[248, 366, 39, 104]	[6, 95, 241, 273, 101, 14]
F'_{37}	[254, 378, 225, 100]	[2, 11, 109, 256, 269, 77, 7]
Extension of F'_{37}	[244, 349, 57, 107]	[6, 75, 233, 276, 119, 19, 1]
F'_{38}	[254, 378, 225, 100]	[2, 11, 109, 256, 269, 77, 7]
Extension of F'_{38}	[244, 349, 57, 107]	[6, 75, 233, 276, 119, 19, 1]
F'_{39}	[254, 378, 225, 100]	[2, 11, 109, 256, 269, 77, 7]
Extension of F'_{39}	[244, 349, 57, 107]	[6, 75, 233, 276, 119, 19, 1]
F'_{40}	[225, 378, 28, 126]	[14, 143, 249, 243, 71, 8]
F'_{41}	[225, 378, 28, 126]	[14, 143, 249, 243, 71, 8]
F'_{42}	[225, 378, 28, 126]	[14, 143, 249, 243, 71, 8]
F'_{43}	[202, 369, 31, 155]	[23, 112, 272, 242, 62, 13]
F'_{44}	[202, 369, 31, 155]	[23, 112, 272, 242, 62, 13]
F'_{45}	[202, 369, 31, 155]	[23, 112, 272, 242, 62, 13]
F'_{46}	[202, 369, 31, 155]	[27, 107, 260, 261, 60, 10]
F'_{47}	[202, 369, 31, 155]	[27, 107, 260, 261, 60, 10]
F'_{48}	[202, 369, 31, 155]	[27, 107, 260, 261, 60, 10]
F'_{49}	[185, 351, 34, 187]	[1, 23, 84, 257, 247, 101, 9]
F'_{50}	[185, 351, 34, 187]	[1, 23, 84, 257, 247, 101, 9]
F'_{51}	[185, 351, 34, 187]	[1, 23, 84, 257, 247, 101, 9]
F'_{52}	[174, 324, 37, 22]	[17, 60, 201, 295, 122, 24]
$\bar{F}_{53}$	[345, 343, 18, 51]	[59, 216, 333, 129, 1]
Extension of $\bar{F}_{53}$	[244, 294, 111, 108]	[14, 94, 203, 233, 143, 39, 1]
$\bar{F}_{54}$	[234, 379, 27, 117]	[1, 8, 24, 169, 319, 165, 40, 3]
Extension of $\bar{F}_{54}$	[234, 351, 55, 117]	[8, 24, 169, 319, 165, 40, 3]
$\bar{F}_{55}$	[234, 379, 27, 117]	[39, 167, 321, 153, 48, 1]
$\bar{F}_{56}$	[177, 334, 36, 210]	[18, 126, 306, 189, 62, 18, 1]
F'_{57}	[361, 333, 18, 45]	[73, 288, 306, 63, 9]
Extension of F'_{57}	[233, 229, 162, 133]	[5, 28, 116, 186, 212, 129, 39, 7, 1]
F'_{58}	[316, 357, 21, 63]	[18, 156, 312, 205, 45]

Extension of F'_{58}	[251, 273, 132, 101]	[23, 127, 231, 205, 112, 28, 1]
F'_{59}	[316, 357, 21, 63]	[18, 156, 312, 205, 45]
Extension of F'_{59}	[262, 324, 81, 90]	[23, 127, 231, 205, 112, 28, 1]
$\acute{F}_{60}$	[277, 372, 24, 84]	[18, 156, 312, 205, 45]
Extension of F'_{60}	[251, 273, 132, 101]	[23, 127, 231, 205, 112, 28, 1]
F'_{61}	[277, 372, 24, 84]	[3, 42, 213, 295, 153, 27]
Extension of F'_{61}	[262, 324, 81, 90]	[27, 180, 292, 177, 45, 9]
F'_{62}	[277, 372, 24, 84]	[3, 42, 201, 325, 129, 33]
Extension of F'_{62}	[256, 342, 63, 96]	[21, 141, 256, 231, 81]
$\acute{F}_{63}$	[277, 372, 24, 84]	[3, 42, 201, 325, 129, 33]
Extension of F'_{63}	[256, 342, 63, 96]	[21, 141, 256, 231, 81]
F'_{64}	[277, 372, 24, 84]	[3, 42, 213, 295, 153, 27]
Extension of F'_{64}	[262, 324, 81, 90]	[27, 180, 292, 177, 45, 9]
F'_{65}	[277, 372, 24, 84]	[3, 42, 213, 295, 153, 27]
Extension of F'_{65}	[262, 324, 81, 90]	[27, 180, 292, 177, 45, 9]
F'_{66}	[277, 372, 24, 84]	[3, 42, 201, 325, 129, 33]
Extension of F'_{66}	[256, 342, 63, 96]	[21, 141, 256, 231, 81]
F'_{67}	[244, 378, 27, 108]	[6, 81, 202, 294, 132, 15]
F'_{68}	[244, 378, 27, 108]	[6, 81, 202, 294, 132, 15]
F'_{69}	[244, 378, 27, 108]	[6, 81, 202, 294, 132, 15]
F'_{70}	[217, 375, 30, 135]	[3, 6, 64, 261, 255, 117, 21]
F'_{71}	[217, 375, 30, 135]	[12, 81, 213, 282, 127, 12]
F'_{72}	[217, 375, 30, 135]	[3, 6, 64, 261, 255, 117, 21]
F'_{73}	[217, 375, 30, 135]	[3, 6, 64, 261, 255, 117, 21]
F'_{74}	[217, 375, 30, 135]	[12, 81, 213, 282, 127, 12]
F'_{75}	[217, 375, 30, 135]	[12, 81, 213, 282, 127, 12]
F'_{76}	[196, 363, 33, 165]	[3, 84, 225, 253, 135, 24]
F'_{77}	[196, 363, 33, 165]	[3, 84, 225, 253, 135, 24]
F'_{78}	[196, 363, 33, 165]	[3, 84, 225, 253, 135, 24]
F'_{79}	[181, 342, 36, 198]	[9, 51, 193, 276, 159, 33]
F'_{80}	[181, 342, 36, 198]	[9, 51, 193, 276, 159, 33]
F'_{81}	[181, 342, 36, 198]	[9, 51, 193, 276, 159, 33]
F'_{82}	[181, 342, 36, 198]	[72, 171, 288, 171, 10, 9]

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